The yield stress of opposed anvils

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The Lur'e solution for an elastic cone has been integrated to give a more reasonable solution of the state of stress of a Bridgman anvil. The maximum shear stress at the center axis is plotted as a function of the tapered angle. It shows that with a smaller tapered angle the anvil gives higher pressure before it plastically yields. It is estimated the maximum pressure can be achieved in a Drickamer-type apparatus with pistons made of maraging steel is around 85 kbar and that of cemented tungsten carbide is around 300 kbar. Based on published claims of achieved pressures, the maximum pressure capability of sintered-diamond compact is greater than 1.2 Mbar and that of a single-crystal diamond could possible be as high as 3.2 Mbar.

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I. INTRODUCTION

The yield strength of a Bridgman anvil is an important parameter in estimating the maximum pressure that can be achieved in a Bridgman opposed anvil apparatus or a standard Drickamer-type apparatus. Unfortunately, the materials for making these pistons, such as grade 999 Carboloy$^{\mathrm{T}}$ cemented tungsten carbide, sintered diamond, and single-crystal diamond all fracture in a brittle fashion with negligible plastic deformation. Therefore, the yield strengths of these materials at room temperature are usually unknown. However, the tip of the Bridgman anvil, while under compression with proper support, provides a sufficiently large hydrostatic stress component such that the plastic deformation of these materials is possible. Recently, Ruoff and Wanagel[1] by approximate analysis of the state of stress in supported opposed anvils and by the measurement of the pressure at which the anvil tips exhibit a permanent deviation from planarity were able to estimate the yield stress of the cemented tungsten carbide. It is shown in the present paper that the Lur'e solution[2] they had used can be integrated to give a more reasonable solution of this problem; the maximum shear stress at the center axis can be obtained as a function of the tapered angle of the anvil. In Sec. III the maximum pressure achievable with pistons made of different materials is discussed.

II. THEORETICAL

The standard Drickamer-type apparatus consists of two opposed tungsten carbide pistons placed in a cylinder with the space between them filled with pyrophyllite. The Bridgman opposed anvil apparatus is more or less the same; the difference is that there is no support over the conical flanks of the pistons. The specimen is encased in a circular pyrophyllite ring along with two disks and is placed in between the two anvils. It has been shown by Forsgren and Drickamer[3] that the pressure on the anvil tip is more or less uniform. However, it should be noted that it is only true when the sample thickness (i.e., the gap between the opposed anvils) is very small. It was shown by Myers *et al.*[4] that the pressure distribution over the flat surface of the anvil depends strongly upon the diameter-to-thickness ratio of the sample. Recently, Bundy[5] also showed that the slope of the pressure on the flat surface versus the applied load increases with decreasing gap thickness. Therefore, it can be concluded that in a Drickamer apparatus with a very thin sample, the pressure on the flat is approximately uniform to the edge of the circular flat but then decreases rapidly over the tapered surface. However, for a thick sample, the pressure on the flat has a Gaussian-type distribution.[6] The latter is also true for opposed anvil apparatus. In the present paper, we will consider a situation where the flat surface of radius a of the anvil is subject to a uniform pressure as shown in Fig. 1(a). This approximates the case of a very thin sample. We shall now proceed to find the state of stress inside the anvil.

Lur'e[2] has shown the elastic solution for the case in which a point load is applied at the vertex of a cone and acts inward along the axis of the cone. His solution in spherical coordinates is as follows:

$$\begin{aligned}
\sigma_R &= \frac{C}{R^2}(1+\cos\gamma - A\cos\theta),\\
\sigma_\theta &= \frac{C}{R^2}\left(\frac{\cos\theta(\cos\theta-\cos\gamma)}{1+\cos\theta}\right),\\
\sigma_\phi &= \frac{C}{R^2}\left(\frac{\cos\theta-\cos\gamma}{1+\cos\theta}-1+\cos\theta\right),\\
\sigma_{R\theta} &= \frac{C}{R^2}\left(\frac{\sin\theta(\cos\theta-\cos\gamma)}{1+\cos\theta}\right),
\end{aligned} \tag{1}$$

where

$$C=\frac{Q(m-2)}{8\pi(m-1)},$$

$$Q=\frac{4F(m-1)}{m(1-\cos^3\gamma)-(m-2)\cos\gamma(1-\cos\gamma)},$$

$$A=\frac{4m-2}{m-2},$$

and $m=1/\nu$, ν is Poisson's ratio, γ is the half-apex-angle of the cone, F is the force acting along the cone axis, and R and θ are shown in Fig. 1(b). Now, in order to obtain a solution for the case of Fig. 1(a), we integrate the Lur'e solution[2] with the vertex of the cone tracing out a circular area with the diameter equal to that in Fig. 1(a). This is the same as Timoshenko and Goodier[7] did for the solution of the Boussinesq problem. For convenience in later analysis, we shall transform the stress tensor components to a cylindrical

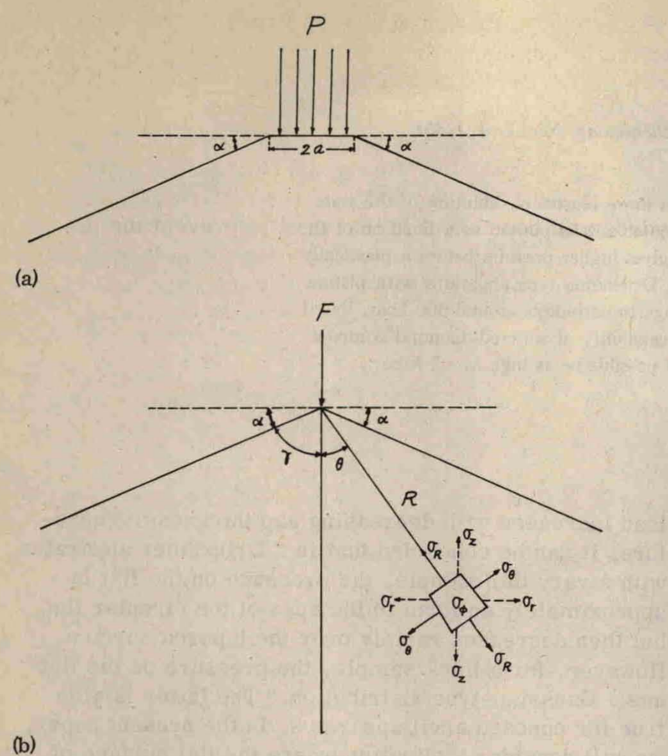

FIG. 1. (a) Side view of anvil with uniform loading. (b) Cone with a point load at the vertex.

coordinates by using the following equations:

$$\begin{aligned}\sigma_z &= \sigma_R \cos^2\theta + \sigma_\theta \sin^2\theta - 2\sigma_{R\theta}\sin\theta\cos\theta,\\ \sigma_r &= \sigma_R \sin^2\theta + \sigma_\theta \cos^2\theta + 2\sigma_{R\theta}\sin\theta\cos\theta,\\ \sigma_\phi &= \sigma_\phi,\\ \sigma_{rz} &= (\sigma_R - \sigma_\theta)\cos\theta\sin\theta + \sigma_{R\theta}(\cos^2\theta - \sin^2\theta).\end{aligned} \tag{2}$$

After integration, we obtain the stress distribution along the z axis as follows:

$$\begin{aligned}\sigma_z &= 2\pi C[-\tfrac{1}{3}(A-1)(1-\cos^3\beta) + \cos\gamma(1-\cos\beta)],\\ \sigma_r &= \sigma_\phi = \pi C[\tfrac{1}{3}(A-1)(1-\cos^3\beta)\\ &\quad + (2 - A - \cos\gamma)(1-\cos\beta)],\end{aligned} \tag{3}$$

where

$$\cos\beta = [z/(a^2+z^2)^{1/2}].$$

When $\gamma = 90°$, this solution becomes exact and reduces to that of Timoshenko and Goodier[7] on the Boussinesq problem, i.e., a uniform pressure acting normally on a circular area of radius a on the planar boundary of a semi-infinite medium. For γ slightly less than 90°, or small tapered angle α ($\alpha = 90° - \gamma$), this solution should be very close to the exact solution.

It is easily shown that the shear stress $\frac{1}{2}(\sigma_\phi - \sigma_z)$ becomes a maximum along the z axis at a depth

$$z = a[k/(1-k)]^{1/2}, \tag{4}$$

where

$$k = \frac{(A + 3\cos\gamma - 2)}{3(A-1)},$$

and the shear stress there is

$$\begin{aligned}&\tfrac{1}{2}(\sigma_\phi - \sigma_z)_{\max}\\ &\quad = \tfrac{1}{2}\pi C[(A-1)(1-\cos^3\beta) + (2 - A - 3\cos\gamma)(1-\cos\beta)].\end{aligned} \tag{5}$$

If we use the maximum yield stress criterion of Tresca, then yielding occurs when $(\sigma_\phi - \sigma_z)_{\max} = \sigma_0$, where σ_0 is the yield strength of the material. With this analytic solution, it enables us to plot the ratio $(\sigma_\phi - \sigma_z)_{\max}/\sigma_{z=0}$ as a function of the tapered angle. It is shown in Fig. 2 for the case of cemented tungsten carbide with $\nu = 0.185$. From the curve, we see that with a smaller tapered angle, one can achieve higher pressure before yielding. However, with a very small angle (a flat being the limit) and a very small circular flat, one should note that the increase of contact area due to elastic deformation of the tip would limit the maximum pressure geometrically.

From the solution we obtained, we see that σ_z, σ_r, and σ_ϕ are all negative at the tip of the anvil. σ_r and σ_ϕ are only slightly less in magnitude than σ_z; because of this, there is a large hydrostatic component at the tip which prevents the material from brittle fracture and also makes the plastic deformation possible.

III. DISCUSSION

Now, we shall assume a standard configuration of a Drickamer-type apparatus. i.e., tapered angle $\alpha = 18°$, and proceed to calculate the maximum pressure before yielding for anvils of different materials. We shall estimate the maximum pressure obtainable with opposed anvils made of maraging steel, cemented tungsten carbide, single-crystal diamond, and sintered diamond.

For maraging steel, we have $\nu = 0.30$. When substituted in Eqs. (3)–(5), we obtain $\sigma_0 = 0.666\sigma_{z=0}$ and yielding starts at $z = 0.702a$. With the known yield stress for maraging steel equal to 20 kbar, we would expect the pressure at the onset of the plastic deformation at around 30 kbar.

For cemented tungsten carbide, we use the value $\nu = 0.185$ obtained from elastic constant measurement

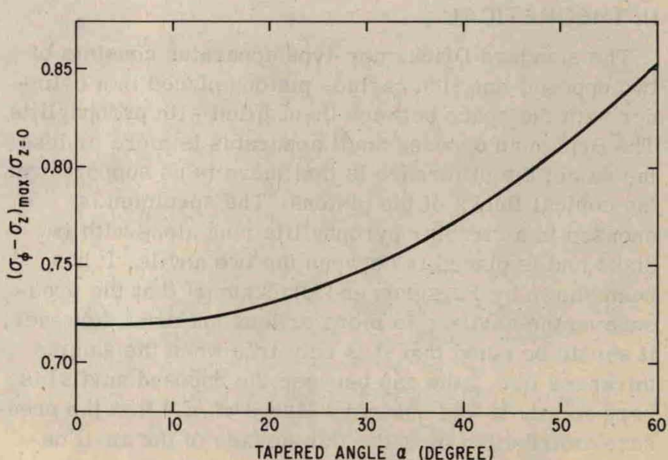

FIG. 2. The ratio of $(\sigma_\phi - \sigma_z)_{\max}$ to σ_z at $z=0$ as a function of tapered angle α.